identifier is used to distinguish each record. References are constituted by bibliographical information on theorems and conjectures, and pictures of special digraphs, which are accessed through hyperlinks by the GRAPHLOG external module [5]. Comments are textual complementary information that can be introduced by the user, such as a date or 'best result'. The text field contains data on the theorem, invariant relation or digraph example. This information is written in LEC and, in case of modification, it is syntactically and semantically verified before being translated to the PROLOG internal form understood by ARS. Theorems, Invariant Relations and Examples of Digraphs are further described in Section 3.

The Help System is a standard on-line help tool that can be accessed from any of the AUI modules.

Finally, the Interface Manager controls the main modules of the AGORA Interface System.

3. The Reasoning System

In what follows, the main characteristics of ARS will be presented (see Figure 2).

![Figure 2. Architecture of the reasoning system](image)

3.1 The ARS architecture

ARS is constituted by three modules:

1. The AKB, partitioned into three interacting submodules IR, EX and TH, is based on article [3] and it contains: 30 integer invariants, 23 boolean invariants, 3 integer invariants for bipartite digraphs, 10 boolean invariants for bipartite digraphs, 119 relations between invariants, 136 Theorems and open conjectures, 64 example of digraph families.

   - IR: the base containing relations between digraph invariants. There are two types of relations between invariants in IR called (absolute) invariant relations which are sets of basic constraints and (conditional) invariant relations which are conditional constraints. The following are examples of these relations:

     \[
     \begin{align*}
     \text{alpha0} & \leq \text{alpha1} \leq \text{alpha2}. \\
     \text{hamiltonian} & \Rightarrow 1 \text{ connected}. \\
     \text{k connected and } k+1 & \Rightarrow (k-1) \text{ connected}. \\
     \text{k connected} & \Rightarrow \text{minimum} \geq 2k. \\
     \text{k connected} & \Rightarrow \text{minimum} \geq k. \\
     \text{k connected} & \Rightarrow \text{minimum} \geq 2k. \\
     \text{minimum} & \geq \text{nodes}/2 \Rightarrow 1 \text{ connected}. \\
     \text{symmetric} & \Rightarrow \text{alpha1}=\text{alpha2}. \\
     \text{k connected} & \Rightarrow \text{nodes} \geq k+1. \\
     \text{woodall} & \Rightarrow \text{nodes} \Rightarrow 1 \text{ connected}. \\
     \text{meyniete} & \Rightarrow 2 \text{ woodall}. \\
     \end{align*}
     \]

   - EX: the base containing examples of digraphs and of digraph families descriptions; each example is specified as a list of invariants values that the digraph or digraph family must satisfy.

\[5\] This subsystem, part of the GReAt environment [16], can run also as a stand-alone system and implements a hypertext library and a digraph database, on graph theory specialized knowledge.
- TH: the base containing digraph theorems, represented as conditional constraints; each theorem is specified as $H \Rightarrow T$, where $H$ and $T$ are descriptions of digraph families. $T$ can also be a disjunction of digraph families descriptions. Some examples of theorems are:

$\text{woodall} \geq \text{nodes} \Rightarrow \text{hamiltonian}$.

$k \geq 2$ and $\alpha_2 \geq (\alpha_1 + 2)$

$2 \geq \alpha_2$ and $(\text{nodes} \geq 2h + 1)$ and

$\left(\text{minimum} \geq h\right) \Rightarrow \text{circumference} \geq 2h$.

2. The deductive mechanism, based on the Subsumption Algorithm [9]. It constitutes the ARS inference machine and accomplishes the refutation and proof processes of the conjecture to be evaluated. For evaluation purpose, this mechanism uses also the information present in AXB.

3. The explanation subsystem.

Once a conjecture is proved or refuted, explanations are given to the user on the evaluation process followed by ARS, justifying logically the solution obtained. The main feature of this module is to use the information produced by the deductive mechanism while proving or refuting a conjecture, in order to construct the explanation. This information is stored in a temporary text file and is retrieved by the CE for displaying the explanation to the specialist.

3.2 Potentiality of AGORA through examples.

In this section, some examples will be given, in order to focus on the AGORA potentiality. The ARS functioning is the following:

Consistency checking process.

The input to ARS is a conjecture of the form $H \Rightarrow T$. In ARS, conjectures are firstly submitted to a consistency checking process, to prove that the digraph family defined by $H$ is not empty. That is to say, it exists a digraph satisfying the relations between invariants specified in $H$. If this family is empty, it is not consistent, then the conjecture is considered also not consistent. In this case the ARS deductive mechanism will not evaluate the conjecture and it will inform the user of the inconsistency found.

Refutation process.

After the consistency checking, ARS will try to refute it. The refuting process is accomplished by the deductive mechanism, by means of the information contained in EX. Basically, this process try to find a counterexample D that causes the conjecture to be false. D will be a digraph or digraph family which does not satisfy $T$ and satisfies the relations between invariants specified in $H$.

Proof process.

If the refuting process fails, the system will prove the conjecture using the knowledge stored in IR and TH. The proof, according to the Subsumption Algorithm [9], is accomplished as follows:

1. Expansion of $H$ using the relations among the invariants in IR. The expansion of $H$ used by the Subsumption Algorithm [9], contains only constraints which appeared originally in $H$ plus the constraints entailed by $H$ under IR. It will be denoted by $\text{expansion}(H)$. For example, given $H=\text{symmetric}$ and using the invariant relation $\text{symmetric} \Rightarrow \alpha_0=\alpha_1=\alpha_2$ we have $\text{expansion}(H)=\text{symmetric}$ and $\alpha_0=\alpha_1=\alpha_2$.

2. Verify if the conjecture (with expanded $H$) can be established as true from relations between invariants in IR. That is to say, check that $T$ is satisfied in $\text{expansion}(H)$. If this is the case, the process ends successfully.

3. Proof the conjecture using theorems in TH. This process selects a theorem or relation $H' \Rightarrow T'$. If the thesis of the theorem or relation is the same as the conjecture thesis, then we try to prove that $\text{expansion}(H)$ is subsumed by $H'$. The following strategy is applied: