A TOOL FOR CONJECTURING IN GRAPH THEORY\textsuperscript{1}

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Abstract

The conjecturing processes in graph theory means all the activities related with the study of a conjecture: solve or prove, reject or leave it open. The main goal of this work is to describe the basic elements constituting the AGORA (A diGraph eConjecture Research Assistant) tool, for helping the graphist in the conjecturing process. The logical kernel of the tool is the reasoning system, based on terminological logic and the constraint logic programming paradigm. The main reasoning capabilities are performed by a subsystem composed fundamentally by a knowledge base constituted by selected examples of digraphs, relations between invariants and theorems in digraph theory, specifically on paths and circuits and a subsumption algorithm for consistency, proving and refutation. An important additional feature of the system is to provide an explanation concerning the conjecture deduction process, through a powerful graphical user interface. The potentiality of AGORA, in the sense of the reasoning capabilities offered by the system, is illustrated through several examples.

Keywords: reasoning systems, conjecturing process, digraph conjecture, deduction mechanism, interconnection networks, explanation, GUI, object-oriented GUI, interface agents, PAC model

1. Introduction and terminology

The conjecturing process in graph theory means all the activities related to the study of a conjecture: solve or prove, reject or leave it open, on the basis of the existing knowledge (relations between invariants, reasoning strategies or methods, existing theorems, open conjectures, digraphs families). Invariants are integer or boolean values that are preserved under isomorphism. The number of nodes and the number of arcs are obvious integer invariants; they are related by arcs = nodes - 1. Some boolean invariants are: hamiltonian (the value true of the boolean variable hamiltonian means that a digraph contains a hamiltonian circuit), traceable (the value true of the boolean variable traceable means that a digraph contains a hamiltonian path); they are related by hamiltonian = traceable.

In graph theory, a conjecture is an implicational statement of the form $H \Rightarrow T$, whose antecedent called (H)ypothesis describes the conjunction of boolean invariants and/or relation between integer invariants that a digraph family may fulfill in order to satisfy another conjunction of invariant relations described by the consequent, called (T)heesis. A very important terminological relation between set descriptions is the subsumption relation: we say that a set description $H$ is subsumed by a set description $T$, if the family of digraphs which satisfies $H$, denoted by $H^f$ is a subset of the family of digraphs which satisfies $T$ denoted by $T^f$. Notice that to prove $H \Rightarrow T$, is equivalency to prove that $H^f$ is subsumed by $T^f$.

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