point to the particle, so that

\[ \vec{VP} = \vec{P} - \vec{V}. \]

Figure 2. The particle distance to a ray.

However, this procedure can produce particles that behave like circles with sharp borders, producing an artifact known as aliasing. To solve this artifact, this paper proposes the introduction of a modulation function for transparency attenuation. The use of this modulation function produces particles with blurred borders, or anti-aliased particles [11].

So, the new equation for computing the ray color is:

\[ C_r = C_r + C_p \cdot T_p \cdot Tr(d) \]

where the modulation function \( Tr(d) \) for particle transparency is defined as

\[ Tr(d) = At \left( \frac{d}{d_{At}} \right)^4. \]

This Gaussian function models the particle color transition according to the particle size. \( At \) and \( d_{At} \) define the shape of the Gaussian curve, where \( At \) defines the attenuation performed in the particle color when it is at a distance \( d_{At} \). Figure 3 shows one example of the definition of a modulation function using \( At \) and \( d_{At} \). When a particle is exactly over the ray, so \( d = 0 \), there is no attenuation and \( Tr(d)=1 \). The main advantage of this modulation function is that it promotes a continuous transition of the particle color according to its distance to a ray.

Figure 3. Modulation function definition.
2.3 Shadowing a Particle System

The third procedure necessary to completely integrate particle system and a ray tracing system is shadowing procedure. In this procedure the ray intensity must be diminished when this ray transverse a particle system. To perform this attenuation, three steps must be considered:

1. The closest and the second closest intersection between the ray and objects must be computed, using the light source as the view point;
2. The cells between the closest and the second closest distance must be transversed and the opacity must be accumulated;
3. After obtaining the opacity between the closest and the second closest distance the illumination must be attenuated according to this factor.

Figure 4 shows some shadowing situations, where $P_1$ and $P_2$ have total shadow (the ray intersects a solid object), $P_3$ has a partial shadow (the ray intersects a particle system contained in the box) and $P_4$ has no shadow (the ray intersects no object).

![Figure 4. Some shadowing situations.](image)

2.4 Reducing Memory Requirements

Another problem to be considered in the integration of particle systems and ray tracing environments is the demand of memory required to store a particle system. For instance, if a particle system has 500,000 particles, storing just position, transparency and color attributes for each particle will require nearly 17 megabytes to store the complete particle system.

This paper suggests the use of a discrete reduced representation of each attribute of a particle system. This representation uses one byte to store the integer part and one byte to store the fractionary part of a floating point number. This reduce the representation of a floating point number that usually is stored in 8 bytes to just 2 bytes.

As demonstrated by Steigleder [11] this reduction process produce no perceptual artifacts on the final rendered image, mainly due to the fact that particle systems naturally have stochastic perturbations.

An alternative method to a more compact representation of a particle system is to use spherical coordinates to represent three-dimensional points and vectors. Using this technique,