PORTFOLIO SELECTION WITH A GENETIC ALGORITHM

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The selection of stocks and bonds to maximize returns and minimize uncertainty of an investment is not an easy task, because the economic forces are not well understood and non-economic influences can change the success of a particular security.

An important feature of security investments is the correlation among security returns. If security returns were not correlated, diversification could eliminate risk. The fact that security returns are highly correlated, implies that diversification can reduce risk but not eliminate it.

The security analyst formulates statements about the future of securities, then the portfolio analyst selects a list of stocks and bonds as a balanced whole, providing the investor with high returns and reducing the uncertainty.

It is assumed that past averages and standard deviations are reasonable indicators of likely return and uncertainty of return in the future. The portfolio analyst portrays the combinations of likely return and uncertainty of return for different portfolios and then determines the portfolio which provides the most suitable combination of risk and return.

There are two basic objectives to all investors:

1. They want a high return.
2. They want this return not subject to uncertainty.

To be efficient a portfolio P must meet the following conditions:

1. If any portfolio has a greater expected return, it must also have a greater variance of return than portfolio P.
2. If any portfolio has a smaller variance of return, it must also have a smaller expected return than portfolio P.

Markowitz explains several computing procedures to find efficient portfolios. This article presents a new approach to portfolio selection using a genetic algorithm.

GENETIC ALGORITHMS

Genetic algorithms are different from other optimization procedures in four ways:

1. GAs work with a coding of the parameter set.
2. GAs search from a population of points.
3. GAs use only the information of the objective function.
4. GAs use probabilistic transition rules.
To apply a genetic algorithm to the portfolio selection problem, it is necessary to code the fractions invested in each security of the portfolio as a binary string (chromosome). Then the genetic algorithm is used to maximize an objective function. This function includes the expected return of the portfolio and the variance of return of the portfolio.

Figure 1 shows the flow diagram of the genetic algorithm. The population is composed of chromosomes (binary strings), with each chromosome representing a portfolio. The chromosomes are composed of genes; a gene representing the fraction invested in each security of the portfolio.

The initial population contains a fixed number of portfolios and evolves by crossing chromosomes that maximize the objective function in each generation. After several generations, each portfolio of the final population must be evaluated according to the efficient portfolio criteria to select the efficient ones.

All the portfolios generated by the genetic algorithm are legitimate, in the sense that each fraction invested (gene) is greater than or equal to zero.

**DERIVATION OF E, V EFFICIENT PORTFOLIOS**

The expected or average return of a portfolio is,

$$ E = \mu^T X $$

the variance of return of a portfolio is,

$$ V = X^T C X $$

and the standard deviation is,

$$ \sigma = \sqrt{V} $$

where,

- $\mu$ : the column vector of expected returns (n securities)
- $X$ : the portfolio is represented by this column vector ($x_i$ is the fraction invested on security $i$)
- $C$ : the matrix of covariances between the n securities
- $T$ : transpose

In other words, the standard deviation of a portfolio is determined by:

(a) the standard deviation of each security
(b) the correlation between each pair of securities
(c) the amount invested in each security
To illustrate the genetic solution of the portfolio selection problem, it is compared with a classical solution called the "critical line method" for a particular problem.

Example: If the securities of three companies are characterized by the expected returns,

\[
\mu = \begin{bmatrix} 0.062 \\ 0.146 \\ 0.128 \end{bmatrix}
\]

and the covariance matrix,

\[
C = \begin{bmatrix} 0.0146 & 0.0187 & 0.0145 \\ 0.0187 & 0.0854 & 0.0104 \\ 0.0145 & 0.0104 & 0.0289 \end{bmatrix}
\]

find the efficient portfolios.

Solution: Figure 2 shows the solution with the critical line method. The vertical axis is the standard deviation of the portfolio, and the horizontal axis is the expected return of the portfolio.

The objective function to maximize with the genetic algorithm is,

\[
f = E + \frac{1}{V}.
\]

Figure 3 shows the solution for the conditions:

- population size: 50 portfolios
- size of the gene: 5 bits
- number of generations: 1000

Each point represents a portfolio with expected return \( E \) and standard deviation \( \sigma \). The curve defines the efficient portfolios.

The Figure 4 shows the genetic portfolios for the conditions:

- population size: 50 portfolios
- size of the gene: 10 bits
- number of generations: 1000

Again, the curve defines the efficient portfolios.

An increase in the number of bits of each gene increases the resolution of the answer, but also increases the CPU execution time.
Two efficient portfolios are, 

P1:
Investment in security 1 : 44.88 %
Investment in security 2 : 00.90 %
Investment in security 3 : 54.22 %

Portfolio average return : 0.0985
Portfolio standard deviation : 0.1369

P2:
Investment in security 1 : 61.45 %
Investment in security 2 : 00.76 %
Investment in security 3 : 37.79 %

Portfolio average return : 0.0876
Portfolio standard deviation : 0.1289

Security 2 has the higher expected return (0.146), but also has the higher standard deviation (0.2922). The efficient portfolio maximizes the global expected value (E) and minimizes the global standard deviation of the return (\sigma).

SUGGESTED READINGS


FIGURE 1. Flow diagram of the genetic algorithm.
FIGURE 2. Efficient portfolios with the critical line method.